

Fixing the EW scale in supersymmetric models after the Higgs discovery.

D. M. Ghilencea^{a, b, *}

^a CERN - Theory Division, CH-1211 Geneva 23, Switzerland.

^b Theoretical Physics Department, National Institute of Physics
and Nuclear Engineering (IFIN-HH) Bucharest MG-6 077125, Romania.

Abstract

TeV-scale supersymmetry was originally introduced to solve the hierarchy problem and therefore *fix* the electroweak (EW) scale in the presence of quantum corrections. Surprisingly, the numerical methods that evaluate the likelihood (or $\chi^2 \equiv -2 \ln L$) to fit the experimental data and test the SUSY models do not account for *fixing* the EW scale. When this *constraint* is implemented, the likelihood (or χ^2) receives a significant correction ($\delta\chi^2$) that worsens the data fits of the models. We estimate this correction for the models: constrained MSSM (CMSSM), models with non-universal gaugino masses (NUGM) or higgs soft masses (NUHM1, NUHM2), the NMSSM and the general NMSSM (GNMSSM). Except the GNMSSM model and for a higgs mass $m_h \approx 126$ GeV, one finds that in these models $\delta\chi^2/n_{df} \geq 1.5$, which violates the usual condition of a good fit already before fitting the observables, other than the EW scale itself (n_{df} =number of degrees of freedom). Given its significant (negative) implications for SUSY models, it is suggested that future data fits properly account for this effect, if one remains true to the original goal of SUSY.

*E-mail address: dumitru.ghilencea@cern.ch

1 A correction to the likelihood from fixing the EW scale.

The main motivation for introducing low-energy (TeV-scale) supersymmetry (SUSY) was to solve the hierarchy problem and therefore *fix* the electroweak (EW) scale which remains stable under the addition of quantum corrections up to the Planck scale. This is achieved without a dramatic tuning of the parameters specific to the Standard Model (SM) [1]. SUSY is now under intense scrutiny at the LHC and the recent Higgs-like particle discovery of mass m_h near 126 GeV [2] brings valuable information on the physics beyond the SM. In this work we consider popular SUSY models and examine the impact on their precision data fits of the recently measured m_h combined with the requirement of *fixing* the EW scale (vev v or m_Z) that SUSY was supposed to enforce, when it was introduced.

Current likelihood methods to fit the experimental data and test SUSY models at the LHC involve a remarkable amount of technical expertise and work, backed by impressive computing power. Surprisingly, there is one important problem that these numerical methods overlook when fitting the EW data. More exactly, the current likelihood to fit the data or its $\chi^2 \equiv -2\ln L$ do not account for the χ^2 “cost” of *fixing* the EW scale to its known value, that motivated the idea of supersymmetry. Using our previous [3], we show that this leads to an underestimate of the overall value of χ^2/n_{df} in all popular models used at the LHC. Here n_{df} is the number of degrees of freedom of the model, defined as the number of observables fitted (n_O) minus that of the parameters of the model (n_p). The models analyzed include MSSM-like with different boundary conditions for gaugino and higgs soft masses, NMSSM and a generalized version of it, the so-called GNMSSM.

For a realistic model, a good χ^2 fit is expected to give a ratio¹

$$\chi^2/n_{df} \approx 1, \quad \text{where} \quad n_{df} \equiv n_O - n_p. \quad (1)$$

In this work we show that in all SUSY models, the likelihood (or its χ^2) receives a correction due to the condition of *fixing* the EW scale to the measured value (m_Z^0), that worsens the current data fits. We show how to compute this correction (hereafter denoted $\delta\chi^2$) in the presence of the EW minimum constraints of the Higgs potential. The value of $\delta\chi^2$ depends strongly on the value of the higgs mass m_h . Using the recent LHC result $m_h \approx 126$ GeV, we find in the models other than the GNMSSM, a correction $\delta\chi^2/n_{df} > 1.5$ without including the usual χ^2 cost due to fixing observables other than the EW scale itself. Therefore, under the assumption of a simultaneous minimization of both $\delta\chi^2$ and the “usual” χ^2 (assumed to respect $\chi^2/n_{df} \approx 1$) these models have a total $(\chi^2 + \delta\chi^2)/n_{df} > 2.5$, hardly compatible with the data (in the GNMSSM $(\chi^2 + \delta\chi^2)/n_{df} \approx 2$).

Given its implications for these models and for SUSY in general, it is thus suggested that this correction be included in future data fits. In the following we substantiate these claims and analyze the consequences for the viability of popular SUSY models.

¹ For a set of observables O_i , one defines $\chi^2 = \sum_i (O_i^{th} - O_i^{exp})^2/\sigma_i^2$.

2 The calculation of the correction to χ^2 .

Let us briefly review the numerical calculation of the likelihood in a SUSY model (see for example [4, 5]). One usually chooses a set of observables \mathcal{O}_i well measured such as: the W-boson mass, the effective leptonic weak mixing angle θ_{eff}^{lep} , the total Z-boson decay width, the anomalous magnetic moment of the muon, the mass of the higgs (m_h), the dark matter relic density, the branching ratios from B-physics, $B_s - \overline{B}_s$ mass difference and also additional bounds (not shown) which should be counted in n_{df} , too:

$$m_W, \sin^2 \theta_{eff}^{lep}, \Gamma_Z, \delta a_\mu, m_h, \Omega_{DM} h^2, \\ BR(B \rightarrow X_s \gamma), BR(B_s \rightarrow \mu^+ \mu^-), BR(B_u \rightarrow \tau \nu), \Delta M_{B_s}, \text{ etc.} \quad (2)$$

Note that fixing the EW scale to its accurately measured value (m_Z^0) is not on this list, even though this motivated SUSY in the first place. For each observable \mathcal{O}_i the corresponding probability is often taken a Gaussian $P(\mathcal{O}_i|\gamma, y)$ where by γ we denote the set of SUSY parameters that define the model, while by y we denote nuisance variables such as Yukawa (of top, bottom, etc) and other similar couplings. One then assumes that the observables are independent and multiplies their probability distributions to obtain a total distribution, which regarded as a function of γ, y (with “data” fixed), defines the likelihood L :

$$L = \prod_j P(\mathcal{O}_j|\gamma, y); \quad \gamma = \{m_0, m_{1/2}, \mu_0, m_0, A_0, B_0, \dots\}; \quad y = \{y_t, y_b, \dots\} \quad (3)$$

in a standard notation for the SUSY parameters, that are components of the set γ . To work with dimensionless parameters, all γ should be “normalized” to some scale (e.g. the EW scale $v_0 \equiv 246$ GeV). In the “frequentist” approach one maximizes L or equivalently minimizes the value of χ^2 that, under a common assumption of Gaussian distributions, is defined as

$$\chi^2 = -2 \ln L. \quad (4)$$

One then seeks a good fit, such that $\chi^2/n_{df} \approx 1$ at the minimum, by tuning γ, y to fit \mathcal{O}_i . So, under the current view, one is *tuning* γ, y to fit \mathcal{O}_i and this brings a contribution to the χ^2/n_{df} “cost” in (4), but *tuning* the same γ, y to fix the EW scale does *not* bring a contribution to χ^2/n_{df} , it only brings a “fine” tuning! Such distinction made between the χ^2 “costs” of these tunings is inconsistent².

In other approaches like the “Bayesian” method one further combines L with priors (probabilities for γ, y) to obtain the posterior probability density [4, 6, 7, 8]. In this method one searches for the point with the largest probability in parameter space, given the data.

While the observables \mathcal{O}_i are independent, the SUSY set of parameters γ used to fit them, are not. They are usually constrained (correlated) by the EW minimum conditions.

² Also there is no technical difference or reason to treat separately these two “tunings”.

Indeed, in MSSM-like models, there are two minimum conditions of the scalar potential³; one of them is determining the EW scale v as a *function* of the parameters γ of the model (it is not fixing v to any numerical value). In practice however, when doing data fits, one usually replaces v *by hand*, by the measured mass of Z boson (m_Z^0) and solves this minimum condition for a SUSY parameter instead (usually μ_0). Doing so misses the impact on L of the distribution fixing the observable $m_Z \propto v$ to its measured m_Z^0 , in the presence of the EW minimum constraints. The difference is important. The second minimum condition is fixing one parameter (say $\tan \beta$) as a function of the remaining⁴ γ .

Let us then *fix* the electroweak scale in the presence of the two minimum conditions of the scalar potential for MSSM-like models and evaluate the total likelihood. To fix the notation, the potential is

$$\begin{aligned} V = & m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - (m_3^2 H_1 \cdot H_2 + h.c.) + (\lambda_1/2) |H_1|^4 + (\lambda_2/2) |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 \\ & + \lambda_4 |H_1 \cdot H_2|^2 + [(\lambda_5/2)(H_1 \cdot H_2)^2 + \lambda_6 |H_1|^2 (H_1 \cdot H_2) + \lambda_7 |H_2|^2 (H_1 \cdot H_2) + h.c.] \end{aligned} \quad (5)$$

then denote

$$\begin{aligned} \lambda & \equiv \lambda_1/2 \cos^4 \beta + \lambda_2/2 \sin^4 \beta + (\lambda_3 + \lambda_4 + \lambda_5)/4 \sin^2 2\beta + (\lambda_6 \cos^2 \beta + \lambda_7 \sin^2 \beta); \\ m^2 & \equiv m_1^2 \cos^2 \beta + m_2^2 \sin^2 \beta - m_3^2 \sin 2\beta, \end{aligned} \quad (6)$$

λ is the *effective* quartic higgs coupling and m^2 is a combination of the higgs soft terms⁵.

To compute the overall likelihood, hereafter denoted $L_w(\gamma, y)$, that also accounts for the effect of fixing the EW scale, we use the results of [3] (see also [6]) that we adapt here to a form amenable to the calculation of the profile likelihood⁶. To find $L_w(\gamma, y)$ it is useful to write the two EW minimum conditions as Dirac delta of $f_{1,2}$ defined below

$$\begin{aligned} f_1(\gamma; y, v, \beta) & \equiv v - (-m^2/\lambda)^{1/2}, \\ f_2(\gamma; y, v, \beta) & \equiv \tan \beta - \tan \beta_0(\gamma, y, v), \end{aligned} \quad (7)$$

β_0 is the root of the second minimum condition; $\beta_0, f_{1,2}$ depend on the arguments shown; λ, m^2 also depend on γ, y, β . Taking account of constraints (7) the total (“constrained”) likelihood $L_w(\gamma, y)$ that also accounts for fixing the EW scale is

$$\begin{aligned} L_w(\gamma, y) & = m_Z^0 \int dv d(\tan \beta) \delta[f_1(\gamma; y, v, \beta)] \delta[f_2(\gamma; y, v, \beta)] \delta(m_Z - m_Z^0) L(\gamma; y, v, \beta) \\ & = v_0 L(\gamma; y, v_0, \beta_0(\gamma, y)) \delta[f_1(\gamma; y, v_0, \beta_0(\gamma, y))] \end{aligned} \quad (8)$$

³There is an extra condition in the case of NMSSM and GNMSSM models.

⁴One can choose another parameter (instead of $\tan \beta$) like B_0 , but the effect is just a change of variables.

⁵ The EW min conditions are $v^2 = -m^2/\lambda$, $2\lambda(\partial m^2/\partial \beta) - m^2(\partial \lambda/\partial \beta) = 0$. See also [3, 9].

⁶ The profile likelihood is found by maximizing the likelihood wrt nuisance parameters y , for fixed γ 's.

Here $v_0 = 246$ GeV, $m_Z^0 = g v_0/2 \approx 91.2$ GeV, $m_Z = g v/2$, $g^2 = g_1^2 + g_2^2$ with $g_{1,2}$ couplings for U(1), SU(2). To simplify notation, we did not display the numerical argument v_0 of β_0 , i.e. we denoted $\beta_0(\gamma, y) \equiv \beta_0(\gamma, y, v_0)$. m_Z^0 in rhs compensates the dimension of $\delta(m_Z - m_Z^0)$.

$L(\gamma, y, v, \beta)$ in the rhs of (8) denotes the usual likelihood associated with fitting the observables *other than* the EW scale (m_Z), see eq.(2), before v , $\tan \beta$ are fixed by the EW min conditions. Integrating over v , $\tan \beta$ in the presence of the delta functions $\delta(f_1)$, $\delta(f_2)$ is just a formal way to solve these EW minimum constraints and thus to eliminate these parameters in terms of the rest. We also introduced a $\delta(m_Z - m_Z^0)$ distribution, not imposed in data fits, which fixes the EW scale by enforcing the measured mass of Z boson (m_Z^0) and thus the replacement $v \rightarrow v_0$. Since m_Z^0 is very well measured, using $\delta(m_Z - m_Z^0)$ is indeed justified (however, the result can be generalized to a Gaussian⁷).

A comment about normalization in (8): one has to make sure that the two constraints ($f_{1,2}$) are indeed normalized to unity wrt the variables over which we integrate them, in this case $\tan \beta$ and v , and above this condition is indeed respected⁸. Therefore we stress that there is *no freedom* for any additional factors to be present in eq.(8).

Eq.(8) tells us that the original likelihood in the rhs, evaluated on the ground state and used in data fits is now multiplied by a Dirac delta evaluated at v_0 , $\delta(f_1(\dots, v_0, \dots))$. Further, one recalls that Dirac delta of a function $f_1(z_i)$, $i = 1, 2, \dots, n$, is related to that of its arguments according to a relation (formally derived by a Taylor expansion):

$$\delta(f_1(z_i)) = \delta[f_1(z_i^0) + (\nabla f_1)_0 \cdot (\vec{z} - \vec{z}^0) + \dots] = \frac{1}{|\nabla f_1|_0} \delta[n_j (z_j - z_j^0)], \quad (9)$$

A summation over repeated index j is understood and the subscript “o” of $|\nabla f_1|_0$ means this quantity is evaluated at the point $z_i = z_i^0$ where $f_1 = 0$; n_i are components of the normal \vec{n} to the surface $f_1(z_i^0) = 0$, so $\vec{n} = (\nabla f_1/|\nabla f_1|)_0$. Expression (9) for δ function together with eq.(8) tell us that it is not enough for the parameters of the model to respect the constraint $f_1 = 0$ which is what one actually checks in the numerical fits, and that there is instead an *additional* factor generated, represented by the gradient of the constraint⁹.

The “constrained” likelihood of (8) becomes, after using (9):

$$L_w(\gamma, y) = \frac{\delta[n_i (\ln z_i - \ln z_i^0)]}{\Delta_q(\gamma^0, y^0)} L(\gamma; y, v_0, \beta_0(\gamma, y)), \quad z_i = \{\gamma_j, y_k\} \quad (10)$$

Here γ_j, y_k denote components of the sets γ and y defined in (3) and we used (9) for $\ln z_i$ instead of z_i as variables, to ensure dimensionless arguments for δ function. Δ_q denotes the absolute value of the gradient of f_1 evaluated at a point $z_i^0 = \{\gamma_j^0, y_k^0\}$ that is a solution

⁷ Additional distributions can also be considered for nuisance variables, like top, bottom masses, etc, and assumed to factorize out of L in the rhs, with corresponding integrals over the Yukawa couplings, see [3]; however, these are not considered if one is interested to calculate the profile likelihood.

⁸ $\delta(f_{1,2})$ are just distributions for $v, \tan \beta$ (albeit special ones), so they also must be normalized to unity.

⁹ The gradient measures how the constraint “accommodates” the relative variations of the parameters.

to the EW min condition $f_1(\gamma^0, y^0, v_0, \beta_0(\gamma^0, y^0))=0$, and has the value:

$$\Delta_q^2(\gamma^0, y^0) = \sum_{z_i=\{\gamma_j, y_k\}} \left(\frac{\partial \ln \tilde{v}}{\partial \ln z_i} \right)_o^2 = \sum_{\gamma_j} \left(\frac{\partial \ln \tilde{v}}{\partial \ln \gamma_j} \right)_o^2 + \sum_{y_k} \left(\frac{\partial \ln \tilde{v}}{\partial \ln y_k} \right)_o^2, \quad \tilde{v} \equiv \left(\frac{-m^2}{\lambda} \right)^{\frac{1}{2}} \quad (11)$$

The subscript “o” stands for evaluation on the ground state ($\gamma_i = \gamma_i^0$, $y_k = y_k^0$). Δ_q that emerged above has some resemblance to what is called the fine tuning measure¹⁰ wrt all parameters, both γ and y . The arguments of $\Delta_q(\gamma^0, y^0)$ denote the parameters wrt which is computed and includes SUSY parameters *and* nuisance variables (Yukawa, etc).

Eq.(10) is the result in terms of distributions and can be written in an alternative form. The δ in the rhs of (10) tells us that the lhs is non-zero when $z_i = z_i^0$ for all i (i.e. $\gamma_j = \gamma_j^0$, $y_k = y_k^0$). Another way to present this eq is to do a formal integration of (10) in the general direction $\ln \tilde{z} = n_j \ln z_j$, (sum over j understood, j running over the sets γ, y); this allows all independent parameters to vary simultaneously¹¹. After this, eq.(10) becomes:

$$L_w(\gamma^0, y^0) = \frac{L(\gamma^0; y^0, v_0, \beta_0(\gamma^0, y^0))}{\Delta_q(\gamma^0, y^0)}. \quad (12)$$

L in the rhs is exactly the usual, “old” likelihood computed in the data fits and evaluated at the EW minimum (reflected by its arguments) but *without* fixing the EW scale, while the lhs does account for this effect. Note that due to the minimum condition $f_1(\gamma^0, y^0, v_0, \beta_0(\gamma^0, y^0)) = 0$, one element of the set γ^0 , say γ_κ^0 , becomes a function of the remaining, independent γ_i^0 , $i \neq \kappa$. Usually γ_κ^0 is taken to be μ_0 of (3).

The result in eq.(12) shows that for a good fit in the model (i.e. maximal L_w), one has to maximize not the usual likelihood in the rhs, but actually its *ratio* to Δ_q . Let us introduce the notation $\chi_w^2 \equiv -2 \ln L_w$ and also $\chi^2 \equiv -2 \ln L$, then eq.(12) becomes

$$\chi_w^2 = \chi^2 + 2 \ln \Delta_q. \quad (13)$$

Therefore, after fixing the EW scale the usual χ^2 receives a positive correction that depends on Δ_q and that is not included in the precision data fits. One can go further and compute from (12) the profile likelihood by maximizing the likelihood (minimize χ_w^2) wrt nuisance variables y^0 , for a fixed set of γ^0 . This profile likelihood then takes the form $L_w(\gamma^0, y^0(\gamma^0))$ and can be used in detailed numerical analysis.

Let us remark that in eq.(10) we used $\ln z_i$ as arguments under the Dirac delta function, which implicitly assumes that these are more fundamental parameters than z_i themselves, (here $z_i = \{\gamma_j, y_k\}$). In principle this is a choice, motivated here by the fact that it ensured dimensionless arguments for the delta function in (10), (unlike y_k , γ_j are dimensionful

¹⁰This was introduced in [10], with $\Delta_{max} = \max_\gamma |\partial \ln \tilde{v} / \partial \ln \gamma_i|$; see [11] for an interpretation of $1/\Delta_{max}$.

¹¹Such integral is just a formal way of saying we solve $f_1 = 0$ in favour of one particular γ_κ^0 (usually μ_0).

parameters). Going from these parameters to their log's is a one-to-one change that does not affect the minimal value of total¹² χ^2 . If one insists in working with z_i as fundamental parameters, one simply changes $\ln z_i \rightarrow z_i$ in eqs.(10), (11), after “normalizing” γ_i to some scale (e.g. v_0), to ensure dimensionless arguments for δ of (10). The result is that Δ_q is then computed wrt γ_j and y_k instead of their logarithms, so in eqs.(12), (13) one replaces

$$\Delta_q^2 \rightarrow \Delta_q'^2 = \sum_{\gamma_j} \left(\frac{\partial \ln \tilde{v}}{\partial \gamma_i} \right)_o^2 + \sum_{y_k} \left(\frac{\partial \ln \tilde{v}}{\partial y_k} \right)_o^2 \quad (14)$$

Compared to its counterpart in Δ_q of (11), the second sum above is actually larger in this case since $y_k < 1$. In the following, for numerical estimates we shall work with Δ_q . A detailed investigation of the correction to χ^2 is beyond the purpose of this paper and in the following we restrict the study to a numerical estimate. The main point is that χ^2 receives a correction that needs to be taken into account.

The above discussion can be extended to the Bayesian approach which is just a global version (in parameter space) of (12). In this case one assigns, in addition, some initial probabilities (priors) to the parameters of the model (γ, y) then integrates over them the likelihood L_w multiplied by the priors. This gives the global probability of the model or “evidence”, $p(\text{data})$, that must be maximized. The result is (see eq.(12) in [6] and Section 2.2 in [3]):

$$p(\text{data}) = \int_{f_1=f_2=0} dS \frac{1}{\Delta_q(\gamma, y)} L(\gamma, y, v_0, \beta) \times \text{priors}(\gamma, y) \quad (15)$$

where the integral is over a surface in the parameter space γ, y defined by $f_1 = f_2 = 0$. In this case $1/\Delta_q$ is itself an “emergent”, naturalness prior, independent of and in addition to the original priors of the model. If one makes a change of a subset of variables to a more suitable one for calculations, an additional Jacobian factor emerges under the integral. We do not discuss further the Bayesian approach, for more details see [4, 6, 7, 8, 12, 13, 14].

3 A numerical estimate of the correction to χ^2 in SUSY.

In this section we estimate in some models the correction $\delta\chi^2$ obtained from eq.(13)

$$\delta\chi^2/n_{df} \equiv (2/n_{df}) \ln \Delta_q \quad (16)$$

By demanding that the correction $\delta\chi^2/n_{df}$ be small enough not to affect the current data fits results for χ^2/n_{df} , one has a model-independent upper bound:

¹²In a Bayesian language, this would correspond to choosing log priors instead of flat ones for the parameters. Ultimately this may reflect a problem of *measure* that is beyond the purpose of this work.

$$\Delta_q \ll \exp(n_{df}/2). \quad (17)$$

When this bound is reached, then $\chi_w^2/n_{df} = 1 + \chi^2/n_{df}$, and assuming values of “usual” $\chi^2/n_{df} \approx 1$ i.e. a good fit without fixing the EW scale, gives $\chi_w^2/n_{df} \approx 2$.

Using (16), one can find the equivalent numerical value of Δ_q that corresponds to one observable having a given deviation from the central value. This can be read below:

$$\begin{aligned} 3\sigma &\leftrightarrow \Delta_q \approx 100. \\ 3.5\sigma &\leftrightarrow \Delta_q \approx 1000. \\ 5\sigma &\leftrightarrow \Delta_q \approx 1\,000\,000. \end{aligned} \quad (18)$$

This gives, in a model independent way, a different perspective and a probabilistic interpretation to the numerical values of Δ_q (related to fine-tuning/naturalness) that avoid subjective criteria about this topic. As a result, any model with $\Delta_q > 100$ ($\Delta_q > 200$) is more than a 3σ (3.25σ) away from fixing the EW scale. This is the case of the SUSY models that we discuss next. This effect is not accounted for by current χ^2 fits.

For our numerical estimates of $\delta\chi^2$ we restrict the analysis to using *minimal* values of Δ_q in SUSY models. Further, we only evaluate Δ_q wrt γ parameters; notice that

$$\Delta_q(\gamma, y) > \Delta_q(\gamma). \quad (19)$$

where the arguments display the variables wrt which Δ_q is actually computed. That is, we shall ignore the contribution to Δ_q due to variations wrt Yukawa couplings and other nuisance parameters. This underestimates the correction¹³ $\delta\chi^2$.

We consider the most popular SUSY models used for searches at the LHC, listed below:

- the constrained MSSM model (CMSSM); this is the basic scenario, of parameters¹⁴ $\gamma \equiv \{m_0, m_{1/2}, \mu_0, A_0, B_0\}$, in a standard notation. Then Δ_q is that of (11) with summation over these parameters only:

$$\Delta_q^2 = \sum_{\gamma_j} \left(\frac{\partial \ln \tilde{v}}{\partial \ln \gamma_j} \right)_o^2 \quad (20)$$

For recent, two-loop numerical estimates of Δ_q we use the result in [6, 9].

- the NUHM1 model: this is a CMSSM-like model in which one relaxes the Higgs soft masses in the ultraviolet (uv), to allow values different from m_0 : $m_{h_1}^{uv} = m_{h_2}^{uv} \neq m_0$, with parameters $\gamma \equiv \{m_0, m_{1/2}, \mu_0, A_0, B_0, m_{h_1}^{uv}\}$. Δ_q is as in (20) with summation over this set. For two-loop estimates of Δ_q as a function of the two-loop leading-log m_h we use [6].

¹³Actually, if one integrates (fixes) the masses of the top, bottom, etc, to their numerical values, it is $\Delta_q(\gamma)$ that is finally obtained in eq.(13) [3]. Indeed, integrating in (8) over additional $d(\ln y_t)$ and $d(\ln y_b)$ and with distributions $\delta(m_t - m_t^0)$ and $\delta(m_b - m_b^0)$ to “fix” the accurately measured observables, one obtains a result similar to (10) but no contribution is present in eqs.(11) to (13) from Yukawa couplings [3].

¹⁴Parameters γ are those following from (10), (11) and this is why μ_0 is quoted instead of usual $\text{sgn}(\mu_0)$. Similar for the other models. Also B_0 is quoted instead of $\tan \beta$, the difference is a change of variables.

- the NUHM2 model: this is a CMSSM-like model with non-universal Higgs soft masses, $m_{h_1}^{uv} \neq m_{h_2}^{uv} \neq m_0$, with independent parameters $\gamma \equiv \{m_0, m_{1/2}, \mu_0, A_0, B_0, m_{h_1}^{uv}, m_{h_2}^{uv}\}$. Then Δ_q is that of (20) with summation over this set. For two-loop numerical estimates of Δ_q we use the results in ref. [6].
- the NUGM model: this is a CMSSM-like model with non-universal gaugino masses m_{λ_i} , $i = 1, 2, 3$, with $\gamma = \{m_0, \mu_0, A_0, B_0, m_{\lambda_1}, m_{\lambda_2}, m_{\lambda_3}\}$. Δ_q is that of (20) with summation over this set. For two-loop numerical estimates of Δ_q we use the result in [6].
- the NUGMd model: this is a particular case of the NUGM model with a specific relation among the gaugino masses m_{λ_i} , $i = 1, 2, 3$, of the type $m_{\lambda_i} = \eta_i m_{1/2}$, where $\eta_{1,2,3}$ take only *discrete*, fixed values. Such relations can exist due to some GUT symmetries, like SU(5), SO(10), etc [15]. The particular relation we consider is a benchmark point with $m_{\lambda_3} = (1/3) m_{1/2}$, $m_{\lambda_1} = (-5/3) m_{1/2}$, $m_{\lambda_2} = m_{1/2}$, corresponding to a particular GUT (SU(5)) model, see Table 2 in [15]. Δ_q is that of (20) with $\gamma = \{m_0, m_{1/2}, A_0, B_0, \mu_0\}$. For numerical estimates of Δ_q we use ref. [6].
- the next to minimal MSSM model (NMSSM): the model has an additional gauge singlet. Its parameters are $\gamma = \{m_0, \mu_0, A_0, B_0, m_{1/2}, m_S\}$, with m_S the singlet soft mass. For an estimate of Δ_q we use the results in [16, 17]. Notice that these papers evaluate instead Δ_{max} which is the largest fine tuning wrt to any of these parameters (instead of their sum in “quadrature” as in Δ_q). As a result, Δ_{max} is usually slightly smaller, by a factor between 1 and 2 as noticed for the other models listed above [6] and thus $\delta\chi^2$ is underestimated.
- the general NMSSM (GNMSSM) model: this is an extension of the NMSSM in the sense that it contains a bilinear term in the superpotential for the gauge singlet, MS^2 , see for example [16]. So the singlet is massive at the supersymmetric level¹⁵ and we have an additional parameter M to the set we have for the NMSSM. Again, Δ_{max} is used here [16] instead of Δ_q , so $\delta\chi^2$ is again underestimated (as for the NMSSM).

Our estimates for $\delta\chi^2/n_{df}$ for different higgs mass values are shown in Tables 1 and 2. The results present the *minimal* value of Δ_q evaluated in the above models as a function of the higgs mass¹⁶, after a scan over the entire parameter space (all γ , y and also $\tan\beta$ of the corresponding model, as described in¹⁷ [6]). We show the values of Δ_q for central values of m_h close to the pre-LHC lower bound ≈ 115 GeV in Table 1 and for 123 to 127 GeV in Table 2. These values allow us to account for a 2-3 GeV error of the theoretical calculation at 2-loop leading log level [19]. For $m_h \approx 115$ GeV, one could still have $\delta\chi^2/n_{df} < 1$ for some models, that could have allowed a corrected $\chi_w^2/n_{df} \approx 1$. Further, minimal Δ_q grows approximately exponentially wrt to m_h , due to quantum corrections. Indeed, $\Delta_q \sim m_{susy}^2$ and since the loop correction is roughly $\delta m_h \sim \ln m_{susy}$ one finds $\Delta_q \approx \exp(m_h/\text{GeV})$. As a result, a strong variation of Δ_q wrt m_h is found, and $\delta\chi^2$ increases by ≈ 1 for a 1 GeV increase of m_h , see Table 2.

¹⁵Its mass can be of few (5-8) TeV, so one can integrate it out and work near the decoupling limit [18].

¹⁶ See Figures 1 to 8 in [6] showing Δ_q as a function of m_h , after scanning the entire parameter space.

¹⁷The scan included a range of $[-7, 7]$ TeV for A_0 , m_0 and $m_{1/2}$ up to 5 TeV, and $2 \leq \tan\beta \leq 62$ and also allowed a 2σ deviation for fitted observables and for a 3σ for $\Omega_{DM}h^2$, see for details [6].

Model	n_p	Approx	Δ_q	$\delta\chi^2_{(115)}$	n_{df}
CMSSM	5	2-loop	15	5.42	9
NUHM1	6	2-loop	100	9.21	8
NUHM2	7	2-loop	85	8.89	7
NUGM	7	2-loop	15	5.42	7
NUGMd	5	2-loop	12	4.97	9
NMSSM	6	1-loop	12	4.97	8
GNMSSM	7	1-loop	12	4.97	7

Table 1: The correction $\delta\chi^2 \equiv 2 \ln \Delta_q(\gamma)$, the number of parameters n_p and degrees of freedom n_{df} in SUSY models, for $m_h \approx 115$ GeV corresponding to the pre-LHC bound of m_h . Notice that in this case $\delta\chi^2/n_{df} < 1$. n_{df} may vary, depending on the exact number of observables fitted. The numerical values of Δ_q are from [6] (first 5 models) and [16] for NMSSM, GNMSSM.

Model	Δ_q	$\delta\chi^2_{(123)}$	Δ_q	$\delta\chi^2_{(125)}$	Δ_q	$\delta\chi^2_{(126)}$	Δ_q	$\delta\chi^2_{(127)}$
CMSSM	380	11.88	1100	14.01	1800	14.99	3100	16.08
NUHM1	500	12.43	1000	13.82	1500	14.63	2100	15.29
NUHM2	470	12.31	1000	13.82	1300	14.34	2000	15.20
NUGM	230	10.88	700	13.10	1000	13.82	1300	14.34
NUGMd	200	10.59	530	12.55	850	13.49	1300	14.34
NMSSM	>100	9.21	>200	10.59	>200	10.59	>200	10.59
GNMSSM	22	6.18	25	6.43	27	6.59	31	6.87

Table 2: As for Table 1, with Δ_q and corresponding $\delta\chi^2 \equiv 2 \ln \Delta_q(\gamma)$ for m_h equal to 123, 125, 126 and 127 GeV (shown within brackets). This can also show the impact of the 2-3 GeV error in the theoretical calculation of m_h [19, 20]. Δ_q grows \approx exponentially with m_h [6, 9]; a 1 GeV increase of m_h induces about 1 unit increase of $\delta\chi^2$. In all cases except GNMSSM, fixing the EW scale brings a correction $\delta\chi^2/n_{df} > 1$. Note $\delta\chi^2/n_{df}$ can be larger if one also includes the impact of Yukawa couplings on Δ_q . Same loop approximation, number of parameters and degrees of freedom apply as in Table 1. The values of Δ_q are from [6] and from [16] for NMSSM, GNMSSM.

In all models, for the currently measured $m_h \approx 126$ GeV, $\delta\chi^2/n_{df}$ alone is larger than unity (or close to 1 for GNMSSM), without considering the original contribution due to the “usual” χ^2/n_{df} coming from fitting observables other than the EW scale. The above correction is too large by the usual criteria that total $\chi_w^2/n_{df} \approx 1$. Further, assume that in the above models one finds a point in the parameter space for which $\chi^2/n_{df} \approx 1$ and then add to it the effect of $\delta\chi^2$. One then finds that χ_w^2/n_{df} is close to or larger than 2, see Table 3. In this Table, the values of reduced χ^2 increase (decrease) by ≈ 0.1 for an increase (decrease) of m_h by 1 GeV, respectively, except in the GNMSSM where this is even smaller

$m_h = 126 \text{ GeV}:$	CMSSM	NUHM1	NUHM2	NUGM	NUGMd	GNMSSM
$\chi_w^2/n_{df} :$	2.66;	2.83;	3.05;	2.97;	2.49;	1.94

Table 3: An estimate for total χ_w^2/n_{df} in various SUSY models for $m_h \approx 126 \text{ GeV}$ and $\chi^2/n_{df} \approx 1$.

(0.04 for 1 GeV). The log dependence $\delta\chi^2 = 2 \ln \Delta_q$ means that uncertainties in evaluating Δ_q are reduced and $\delta\chi^2$ values listed in Table 2 and used in Table 3 are comparable, even though corresponding Δ_q 's are very different.

Our estimates for χ_w^2/n_{df} shown in Table 3 are hardly acceptable for a good fit¹⁸. Interestingly, increasing the number of parameters of a model (decrease n_{df}) could decrease Δ_q and $\delta\chi^2$, but this reduction may not be enough to reduce χ_w^2/n_{df} , since n_{df} is now smaller. This is seen by comparing the reduced χ^2 in NUGM and CMSSM. This is because $\delta\chi^2$ depends only mildly (log-like) on Δ_q , so only a significant reduction of Δ_q can compensate the effect of simultaneously reducing n_{df} . For such case compare CMSSM with GNMSSM.

The values of χ_w^2/n_{df} could be higher than our estimates above, since they ignore that: a) we used only minimal values of Δ_q over the whole parameter space. b) Yukawa effects on $\delta\chi^2$ were ignored and these can be significant¹⁹. There is also an argument in favour of a smaller χ_w^2/n_{df} , that current theoretical calculations of m_h may have a 2-3 GeV error. Assuming this, for $m_h \approx 123 \text{ GeV}$ (instead of 126 GeV), in GNMSSM one obtains a small change: $\chi_w^2/n_{df} = 1.8$ while for the other models this ratio is ≥ 2.3 . Another reduction may emerge in numerical analysis if using eq.(14) instead of (11), but the impact of its larger Yukawa contributions (that is usually also larger than that due to γ 's) makes this possibility less likely.

Regarding the NMSSM model, Table 2 only provided a lower bound on Δ_q . However we can do a more accurate estimate, using recent data fits²⁰ that evaluated both χ^2 and Δ_q . To this purpose, we use the minimal value for $\chi^2 = 6.4$ in [17] (last two columns of their Table 3) together with its corresponding²¹ $\Delta_q = 455$. This gives a $\chi_w^2/n_{df} = 18.64/8$, which is similar to the other models discussed above²². Further, using instead our Table 2 where $\delta\chi^2 > 10.59$ we find $\chi_w^2/n_{df} > 2.32$ (for $\chi^2/n_{df} \approx 1$), which is in agreement with the aforementioned value derived from accurate data fits.

To conclude, the requirement of fixing the EW scale has a strong effect on the value of χ_w^2/n_{df} , with negative impact on the data fits and on the phenomenological viability of these models. While our numerical results are just an estimate of the correction $\delta\chi^2$, the effect is nevertheless present and demands a careful investigation by the precision data fits.

¹⁸ Assuming a χ^2 distribution, the p -value in these models would be $< 1\%$ (and 5% for GNMSSM).

¹⁹ In CMSSM, Δ_q wrt top Yukawa alone is larger than that wrt to any SUSY parameter, see fig.2 in [9].

²⁰ For other recent data fits see [21, 22, 23, 24].

²¹ Δ_q is even higher as it is not computed according to (20) but reports max values wrt each parameter.

²² Other values quoted in [17] bring an even larger value for this ratio.

4 Some implications for model building.

Let us discuss some implications of the above result for model building.

a). A natural question is how to reduce $\delta\chi^2/n_{df}$. Here are three ways to attempt this: i) additional supersymmetric terms in the model, which unlike SUSY breaking ones, are less restricted by experimental bounds; ii) additional gauge symmetry, iii) additional massive states coupled to the higgs sector. All these directions have in common a possible increase of the *effective* quartic higgs coupling (λ) so one can more easily satisfy an EW minimum condition²³ $v^2 = -m^2/\lambda$ for $v \sim O(100\text{GeV})$, $m \sim O(1\text{TeV})$, that demands a larger λ . As a result one can obtain a smaller $\delta\chi^2 \propto \ln \Delta_q$. As mentioned, the increased complexity of the model (more parameters) is to be avoided, since then n_{df} can decrease and $\delta\chi^2/n_{df}$ may not change much (or even increase). The GNMSSM model is an example of i), where a supersymmetric mass term for the additional singlet essentially enabled a smaller $\delta\chi^2/n_{df}$ than in other MSSM-like models²⁴. Similar but milder effects exist in the NMSSM²⁵. An increase of λ could also be generated by using idea ii) by adding more gauge symmetry (e.g. [26]). Regarding option iii), one can consider additional massive states that couple to the higgs sector, and that in the low energy generate corrections to the higgs potential and effective λ and m_h [27] with similar effects. These ideas may indicate the direction for SUSY model building.

b). The above negative implications for some SUSY models remind us about the real possibility that no sign of TeV-scale supersymmetry may be found at the LHC. If so, this can suggest its scale is significantly larger than few TeV. Alternatively, one could attempt to forbid the existence of *asymptotic* supersymmetric states while trying to preserve some of the nice advantages of SUSY. In such scenario, superpartners would be present only as internal lines in loop diagrams, they would not be real, asymptotic final states. One could describe this situation by using some variant of *non-linear* supersymmetry that can be described in a superfield formalism endowed with constraints (see examples in [28]). The hope would be to preserve SUSY results like fixing the EW scale, gauge couplings unification, radiative EW symmetry breaking, for which the superpartners in the loops play a crucial role²⁶. However, it is difficult to realize this idea in practice. One also recalls the supersymmetric quantum mechanics case where SUSY is used only as a tool for performing complex calculations [30], which could suggest ideas for the field theory case.

²³ A tension in the relation $v^2 = -m^2/\lambda$ translates into a larger $\delta\chi^2 \sim \ln \Delta_q$ [9]. Recall that in MSSM λ is very small and fixed by gauge interactions (at tree level) and this is one source for the above problems.

²⁴For a large value of the supersymmetric mass term of the singlet (M of few TeV, 5-8), the correction to the higgs effective quartic coupling λ is $\delta\lambda \sim (2\mu/M) \sin 2\beta$ already at tree level [18], with impact on Δ_q and $\delta\chi^2$. [16]. For a recent study of the GNMSSM and its LHC signatures see [25].

²⁵ The correction to λ and m_h is in this case restricted by perturbativity in the singlet coupling $\tilde{\lambda}$ (of $\tilde{\lambda}SH_1.H_2$) and also its proportionality to $\tilde{\lambda} \sin^2 2\beta$ instead, while in the GNMSSM is $\propto \sin 2\beta$.

²⁶A related idea exists [29], based on a similarity to gauge fixing in gauge theories and the subsequent emergence of the ghost degrees of freedom of different statistics. This would have an analogue in the above SUSY scenario in “fixing the gauge” in the Grassmann space. Similar to the emergence of ghosts in gauge theories as non-asymptotic states one could attempt to obtain non-asymptotic superpartners states.

c). The remaining possibility is to abandon (low-energy) SUSY and eventually consider a different symmetry instead. One can use model building based on SM extended with the (classical) scale symmetry, thus forbidding a tree level higgs mass. This symmetry is broken at the loop level by anomalous dimensions, which would bring in only log-like dependence on the mass scales [31]. Additional requirements (unitarity, etc) could be added. For model building along this direction see some examples in [32] and references therein.

5 Conclusions

The main motivation for TeV-scale supersymmetry was to solve the hierarchy problem and therefore *fix* the electroweak scale (vev v or m_Z) in the presence of the quantum corrections. Rather surprisingly, the numerical methods that evaluate the likelihood (or its $\chi^2 \equiv -2 \ln L$) to fit the data in SUSY models do not account for the χ^2 “cost” that is due to *fixing* the EW scale to its measured value (m_Z^0). When this condition is imposed and combined with the constraints of the EW minimum conditions of the higgs sector, one finds that χ^2 receives a positive correction, $\delta\chi^2 > 0$. This correction should be included in the analysis of the total χ^2 of the supersymmetric models.

This result is important, since data fits report separately the result for minimal χ^2/n_{df} that is a measure of *tuning* the parameters to fit some observables (other than the EW scale), and the amount of another *tuning* (a “*fine*” one)²⁷ needed to fix the EW scale, whose χ^2 “cost” is ignored. This suggests an inconsistency. Our results show that these two aspects (“*tunings*”) are actually part of the same problem rather than separate issues and thus the total $\chi^2 + \delta\chi^2$ must be examined by the data fits of the models.

For the recently measured value of the higgs mass (≈ 126 GeV), the correction $\delta\chi^2$ was estimated and was shown to be significant in most popular SUSY models: constrained MSSM (CMSSM), models with non-universal higgs soft masses (NUHM1, NUHM2) or with non-universal gaugino masses (NUGM), in the NMSSM and its generalized version GNMSSM. This correction has negative implications for the data fits of SUSY models. Our estimates show that for $m_h \approx 126$ GeV, this correction alone is $\delta\chi^2/n_{df} > 1.5$, which violates the traditional condition for a good fit already before fitting observables other than the EW scale. Adding this to a good fit according to the “usual” $\chi^2/n_{df} \approx 1$ would give $(\chi^2 + \delta\chi^2)/n_{df} > 2.5$, hardly acceptable. Further contributions to $\delta\chi^2$ also exist from Yukawa couplings, not discussed. The effect is however milder for the GNMSSM case.

To conclude, the requirement of fixing the EW scale in SUSY models brings a correction to the likelihood, with negative implications for the overall data fit and for the phenomenological viability of these models. While we only provided a numerical estimate of it, that needs further investigation, this correction is present and should be included in the total χ^2 , if one remains true to the original goal of SUSY of fixing the EW scale.

²⁷ “measured” instead by various model dependent definitions.

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